Experimental research in noise influence on estimation precision for polyharmonic model frequencies

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Abstract. *Parameter estimating based on measured values for polyharmonic models can be carried out by known three stage method. Its first stage is getting "balancing coefficients" (using linear LS-method); second stage – evaluating harmonic frequencies by solving so-called "frequency polynomial"; third stage – MLS-estimating of harmonic amplitudes and phases. Mentioned scheme gives absolute precise results in case of noise-free data. Noises or measuring errors distort all parameters of harmonic model. Here the most important is the precision of frequencies evaluating, which are derived of "balancing coefficients" via "frequency polynomial", because even small differences in frequencies lead to large inaccuracy in amplitudes and phases. This paper presents experimental comparison of several approximating schemes, that can be used to restore "balancing coefficients" (and so the frequencies), in order to select the most accurate one. It was stated that all examined estimating schemes are almost equivalent under low-noise conditions. One scheme, the approximation based on integrating of difference equations on sliding interval proves statistically the most precise, when noise level is raised.*

Keywords

Polyharmonic model, parameter estimation, noise influence, identification.

Introduction

Polyharmonic models are often used to represent natural cycling process. Polyharmonic models are applicable for analyses of hydrological, geologicalm, meteorological, biological, financial and other time-series haveing cycling behaviour. Such models may be used by itself or as a partial models in GMDH algorythms [1].

While investigating cycling processes parameters of polyharmonic model need to be uncovered from observed time-series. It's desired that corresponding computational procedures were simple enough while being noise-stable in order to gain appropriate model.

Harmonic analyses method and its possible noise-immunity modifications

Polyharmonic model is represented by a sum of single harmonic items having aliquant frequencies

$$Y_k = \sum_{j=1}^{m} \left(A_j \cdot \sin\left(k \cdot \omega_j\right) + B_j \cos\left(k \cdot \omega_j\right) \right), \tag{1}$$

where $k = 0,...,N$ is a discrete time, ω_j is a harmonic frequency, A_j and B_j express amplitude and phase for each of m harmonics.

When identifying parameters of polyharmonic model, the measured values $Y_k^* = Y_k + \xi_k$ are used (where ξ_k is a noise or measuring errors) and the main idea is to minimize the next criterion

$$J(\omega_1,...,\omega_m, A_1,..., A_m, B_1,..., B_m) = \sum_{k=0}^{N} \left(Y_k^* - Y_k\right)^2$$

Direct usage of this criterion with model (1) leads to very complex non-linear optimization problem, which really can not be solved in practice.

There is three-stage method that can simplify parameter estimation for model (1). On the first stage, m additional “balancing coefficients” $\overline{a}_p, p = 0,..., m-1$ are evaluated from the best approximation (in LS sense) of next relations

$$Y_{k+m}^* + Y_{k-m}^* \cong \sum_{p=0}^{m-1} \overline{a}_p \cdot (Y_{k+p}^* + Y_{k-p}^*), \qquad k = m,..., N-m \tag{2}$$

Then calculated “balancing coefficients” are used in equation $\sum_{p=0}^{m-1} \overline{a}_p \cdot \cos(p\cdot\omega) = \cos(m\cdot\omega)$ that is reduced to “frequency polynomial”

$$P_m \cdot \cos^m \omega + P_{m-1} \cdot \cos^{m-1} \omega + ... + P_1 \cdot \cos\omega + P_0 = 0 \tag{3}$$

with m different real roots for m different frequencies $0 < \overline{\omega}_j < 2\pi$, $j = 1,..., m$.

On the third stage amplitudes are defined for all harmonic items from the best approximation (in LS sense) of measured values Y_k^* by model

$$Y_k^* \cong \sum_{j=1}^{m} \overline{A}_j \cdot \sin\left(k\cdot\overline{\omega}_j\right) + \overline{B}_j \cdot \cos(k\cdot\overline{\omega}_j), \quad k = 0,..., N$$

with already evaluated frequencies $\overline{\omega}_j$.

It’s known that described three-stage method leads to absolutely precise estimation of all parameters of polyharmonic model under the assumption of noise-free data ($\xi_k \equiv 0$ for all k). [2] When data are noise-corrupted the “balancing coefficients” (evaluated on the first stage) have values that do not correspond to the valid frequencies (really presented in time-series Y_k). Therefore the second and the third stages will inherit mentioned displacement in values that will be reflected in differences between $\overline{\omega}_i, \overline{A}_j, \overline{B}_j$ and ω_i, A_j, B_j [3]. In this connection the key-point for overall accuracy improvement is achieving better nose-immunity when evaluating “balancing coefficients” at stage one of three-stage method.

There were suggested two modifications of the first stage procedure, which are likely to minimize the influence of noise on “balancing coefficients” when evaluating.

Orthogonal regression method

Interpreting relation (2) as a regression equation we may state that both the output values (left part) and arguments (values in right part) are influenced by noise. So we have a case the orthogonal regression is intended for [4].

Notice that orthogonal regression method is expensive enough in computational sense as it needs to calculate the least eigenvalue and corresponding eigenvector of matrix of normal equations. For not large quantity of harmonics ($m < 10$) we may use special method to calculate the least eigenvalue directly, otherwise the general eigenvalue problem must be solved to select the least one. Therefore this method may be recommended if only it proves the obvious advantage in nose-immunity for “balancing coefficients“.

Integrating of difference equations on “sliding interval”

The main idea of this method may be explained on a sample of simplest discrete dynamic model

$$x_{k+1} = f(x_k, x_{k-1}, ..., x_{k-s})$$

or (in difference form)

$$\Delta x_k = f(x_k, x_{k-1}, ..., x_{k-s}) - x_k .$$

Then the model output at any time $k+1$ may be obtained by “integrating” previous difference equation starting from some discrete time $k-L$ and correspondent initial state x_{k-L}:

$$x_{k+1} = x_{k-L} + \sum_{i=k-L}^{k} \Delta x_i = x_{k-L} + \sum_{i=k-L}^{k} (f(x_i, x_{i-1}, ..., x_{i-s}) - x_i) =$$

$$= \sum_{i=k-L}^{k} f(x_i, x_{i-1}, ..., x_{i-s}) - \sum_{i=k-L+1}^{k} x_i .$$

Here L is the length of the sliding interval.

By analogy with general procedure of integrating on “sliding interval” relations (2) for “balancing coefficients” evaluating may be rewritten in the next form

$$\sum_{i=k-L}^{k} \left(Y_{i+m}^* + Y_{i-m}^* \right) \cong \sum_{p=0}^{m-1} \overline{a}_p \cdot \sum_{i=k-L}^{k} (Y_{i+p}^* + Y_{i-p}^*)$$

For noises ξ_k with zero mean the sum operations in left and right parts of last equation should contribute to decreasing of noise influence on “balancing coefficients” values.

Numerical experiments

There was maid a number of experiments to compare the accuracy of polyharmonic model identification by means of basic method and its two modifications. Time-series for experiments were generated by model (1) with independent uniformly distributed noise. Noise level varied from 0 to 10% of pure polyharmonic signal level. Experiments were carried out with single and two harmonic components with “slow” and “fast” frequencies. Also experiments were repeated with long and short time-series.

To check the consistency of estimation (or exactly the difference between estimated and real frequencies $\Delta\omega_j = \overline{\omega}_j - \omega_j$) procedure of frequency evaluation was held repeatedly for the same model with diferent noise realizations. Average square error in frequencies estimation

$$\Delta(S) = \frac{\sum_{n=1}^{S} \sqrt{\sum_{j=1}^{m} \Delta\omega_j^2}}{S}$$

was calculated while S growth. To gain a a steady ΔS value the number of noise realizations S raised up to 1000.

Some typical results of numerical experiments are presented in Tab.1 and Tab.2.

Behavior of ΔS as function of S is shown below on Fig.1 and Fig.2 for different noice level. Notice that in low noise case LS-method and orthogonal regression give very near results, while integrating on "sliding interval" leads to much more precise frequencies.

Tab.1. Typical dependency of estimation error on frequency

(noise level 5%, time series length 100).

In all experiments "sliding interval" length $L = 4$ there was used.

Real frequency	**Estimate by LS**		**Estimate by orthogonal regression**	
	Frequency	**Error**	**Frequency**	**Error**
0.05	0.052009	1.112603	0.050356	0.313319
0.1	0.101210	0.910304	0.100258	0.341675
0.2	0.201732	1.226560	0.201335	0.958326
0.4	0.400057	0.304304	0.399850	0.3130646
0.6	0.600582	0.501084	0.600429	0.412734
0.9	0.269463	0.269463	0.899963	0.273468
1.2	1.200035	0.294561	1.200005	0.293712
1.6	1.599975	0.300337	1.599977	0.300108

Tab.2. Typical dependency of estimation error on noise level

($\omega_1 = 0.7, \omega_2 = 0.4$, time series length 100).

Noise level	**Estimation by LS**		**Estimation by "sliding interval"**	
	ϖ_1 ϖ_2	**Error**	ϖ_1 ϖ_2	**Error**
0.5%	0.7001265800 0.4027570768	0.62408286538	0.7000227122 0.4001571654	0.05188996321
	0.7003121596 0.4014532905	0.38836392504	0.7001085069 0.4002160055	0.08978823565
	0.7003775879 0.4032954795	0.78734147097	0.7001268259 0.4003112239	0.11716300393
2.5%	0.7058363527 0.4551633137	8.68208063079	0.70074644174 0.40248215615	0.77954468429
	0.7088738959 0.4754817748	9.76012166255	0.70009745386 0.40270307734	0.62241961481
	0.7059137619 0.4543798392	8.68619198744	0.70013033082 0.40143134067	0.34432603714
5%	0.7489917282 0.5821464992	23.1823165362	0.70326640318 0.41053965170	3.220677802312
	0.7618563492 0.5964025406	23.5084822637	0.70254548642 0.41298267142	3.348360503642
	0.7568981144 0.5977817311	23.2879573884	0.70176638686 0.41161431985	2.831692160287

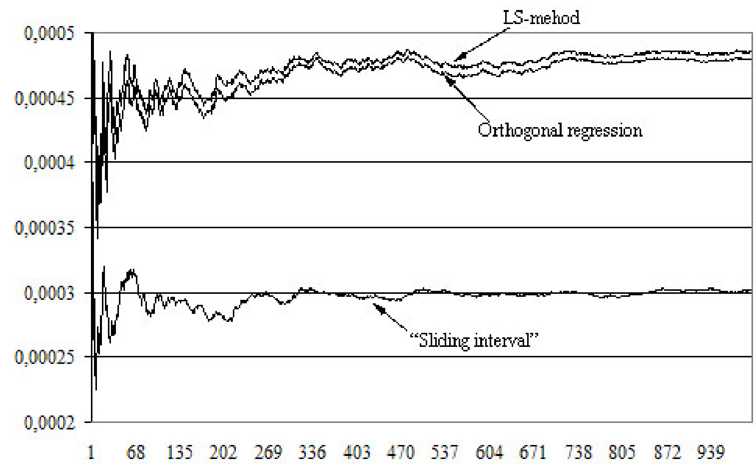

Fig. 1. Average square error in frequencies estimation for three methods (2% noise)

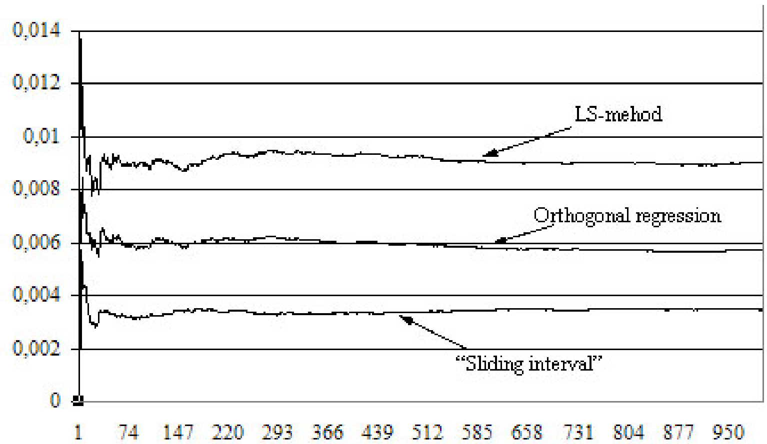

Fig. 1. Average square error in frequencies estimation for three methods (20% noise).

Conclusion

From the numerical experiments we may state the next:

- orthogonal regression method has some advantages in accuracy (in comparison with ordinary LS-method) for "slow" harmonics, for long time-series and in the case of high noise level;
- in all cases the method, which is based on integrating on "sliding interval" demonstrated significantly better accuracy then other two methods;
- it seems that computational complexity of orthogonal regression method neglects its some advantages, especially minding that much more simple method "sliding interval" brings us to better noise-immunity.

References

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