Method of Limiting Generalizations for Solving Logical and Computing Tasks

Yuriy Prokopchuk

Institute of Technical Mechanics of the National Academy of Sciences of Ukraine,
15 Leshko-Popel St., Dnipropetrovsk, 49600, Ukraine.

itk3@ukr.net

Abstract. *The work deals with an efficient method for solution of intellectual logical and computing tasks. The method is based on construction of full knowledge model of the multilevel description of the reality with limiting characteristics. When estimating, the current situation is generalized within the limits proper to the complete model of knowledge. The method corresponds to basic principles of operation of natural intelligence.*

Keywords

Model of knowledge, oriented graph of domains, multilevel description of reality, method of limiting generalizations

1 Introduction

The ease with which man orientates in a complex environment and solves informal problems makes experts in different areas of knowledge study and simulate the natural mechanisms of decision making. An enigma of the natural mechanism is the ability to "automatically" (subconsciously) select only the most important features of the actual situation which are indispensable to the accomplishment of the task facing the individual, these features having the maximum possible level of description (measurement) for the task to be accomplished. The more developed this ability, the more efficient the functioning of natural intelligence [1].

This paper reports a formalism that implements a similar principle of operation in the solution of a wide class of logical and computational problems.

Represent models of the subject domain as the tuple $<O, k>$ where O is the model of ontology of that subject domain, and k is the model of an adequate system of knowledge. Adequacy of the model of the subject domain implies that the model of reality $A(<O, k>)$ coincides with a set of models for every situation including in reality of that subject domain.

Represent the model of knowledge k in a developed view as follows:

$$k = \{f/\mu\colon k^1 \rightarrow k^2\} \cup P_k, \qquad (1)$$

where f/μ is mapping realizing mathematical models in one way or another;
μ are the *d*istinct mechanisms of realization of mapping;
k^1 are the input data of the task (description of information environment and job);
k^2 are the output data of the task;
P_k are the rules of composition of tasks schemas, i.e. the rules describing modes for unifying local tasks.

Consider specifications of tasks for some classes of knowledge models ($\underline{\tau}/T$ are the results of tests; d/D are the conclusions, diagnoses; h/H are the prediction hypotheses; r/R are the control programs; T, D, H, R are the sorts or the domains) [2, 3]:
$F_1 = \{f/\mu\colon \{\underline{\tau}/T\}_1 \rightarrow \{\underline{\tau}/T\}_2\}$ is the class of models for computing knowledge;

$F_2 = \{f/\mu\colon \{\underline{\tau}/T\} \rightarrow d/D\}$ is the class of models of diagnostic knowledge;

$F_3 = \{f/\mu: \{\underline{\tau}/T\} \to \neg d/D\}$ is the class of knowledge models describing the domain of prohibitions;

$F_4 = \{f/\mu: \{\underline{\tau}/T\}, \{d/D\} \to \{h/H\}\}$ is the class of models for prediction knowledge;

$F_5 = \{f/\mu: \{\underline{\tau}/T\}, \{d/D\}, \{h/H\} \to \{r/R\}\}$ is the class of knowledge models for optimization of control;

$F_6 = \{f/\mu: \{\underline{\tau}/T\} \to \{\underline{\tau}/T\}'\}$ is the class of knowledge models for description of the structure and the dynamics of complicated systems represented as collection of causal and consequent relations (both structural ones and time ones).

The general knowledge model k includes all the above-mentioned classes of models, namely:

$F_1 \cup F_2 \cup F_3 \cup F_4 \cup F_5 \cup F_6 \subseteq k.$

The closure of the set of data mapping F^+/P_k is built by means of the rules of composition P_k in solving a specific task [2, 3].

The above schemes of the knowledge model classes F_1 – F_6 illustrate the formal logic level of knowledge representation. Original structures of knowledge representation and ontologies at the procedural level are given in Refs. [3, 10] (by the example of clinical medicine).The basis for the structures is lexical trees [2-4].

The term “domain” has been borrowed from databases, but in the context of this work its treatment is much wider, namely: 1) a domain contains all constructions (linguistic, mathematical and other ones [2]) that make it possible to form the result of a test or conclusion; 2) a domain is certain to involve a semantic situational metric, which, as the situation (the conditions of the problem to be solved) requires, divides the domain into equivalence classes (by meaning). Using semantic metrics, one can establish a correspondence between different domains. At the same time, databases have no metric other than the natural one (character code coincidence). Thus the more complex internal structure of a domain and its semantic metric turn “data and operations on data” into “knowledge and operations on knowledge,” thus extending the treatment of the term “domain” for knowledge bases. This issue is considered in detail in Ref. [2].

2 Method of Limiting Generalizations

Assume that $R^+ = \{\alpha_1, ..., \alpha_m\}$ is the sample of examples with complete information. Suppose that there is a finite set of elementary tests $\{\tau\}$ when any situation of reality α is uniquely re-established from R^+ by values of tests $\{\underline{\tau}\}$.

Assume that one of the tests takes values from finite and alternative sets $D = \{d_1, ..., d_n\}$. Denote that test by τ_d.

Introduce the condition of separability of real situations based on sets of tests $\{\tau/T\}\backslash\tau_d$ and some transitive metric ρ:

$\forall\{\underline{\tau}\}, \{\underline{\tau}\}'$ where $\{\tau\}\subseteq\{\tau/T\}\backslash\tau_d$ and $\exists\alpha, \alpha' \in R^+: \alpha = \alpha(\{\underline{\tau}\},d), \alpha' = \alpha'(\{\underline{\tau}\}', d')$ the following condition should be met: $\rho(\{\underline{\tau}\}, \{\underline{\tau}\}')= 0 \Rightarrow d = d'$.

If a common applied task (the class of applied tasks) may resolved by using in the case a more limited number of notions and statements (theorems), such model of the solution (the model of knowledge) will be considered to be more efficient. Conception of building models of knowledge with a minimal number of objects is dominant for the method under consideration.

Consider the following task.

A task. Assume that a representative sample of real situations R^+ with complete information is given at a particular level of abstraction (the level is determined by domains). Assume that the metric ρ is given in such a manner that the condition of separability is performed on the set R^+. It is required to build a minimal remainder–free model of knowledge on sets R^+ from the point of view of an efficiency function γ: “the classification of the conclusions from D”

In Ref. [3], algorithms for solving the task for different classes of models of knowledge (in the context of a fixed combination of domains) are given. Below are some examples of these classes:

$K_I = \{\{\underline{\tau}/T\} \to d\} \cup \{\neg\{\underline{\tau}/T\}_1 \&...\&\neg\{\underline{\tau}/T\}_m \to\neg d\} \cup (d_1\vee...\vee d_n, \neg d_1 \&...\& \neg d_{n-1} \to d_n).$

$K_{II} = \{\{\underline{\tau}/T\in X_\tau\} \to d\}\cup\{\neg\{\underline{\tau}/T\in X_\tau\}_1 \&...\&\neg\{\underline{\tau}/T\in X_\tau\}_m \to\neg d\}\cup (d_1\vee...\vee d_n, \neg d_1 \&...\& \neg d_{n-1} \to d_n).$

$K_{III} = \cup_{j=1,m} (p_j(\{\underline{\tau}/T\in X_\tau\}_j)=t \to d) \cup \{\&_{j=1,m}\, p_j(\{\underline{\tau}/T\in X_\tau\}_j)=f\to \neg d\} \cup (d_1\vee...\vee d_n, \neg d_1 \&...\& \neg d_{n-1} \to d_n).$

The last mapping in each class is written within the numbering of conclusions. Apart from being the conclusion-making tool, it also serves as the applicability condition.

Let us explain some notations. Let $\{\tau_1, \tau_2, ..., \tau_s\}$ be a set of texts that are involved in the formation of $\{\underline{\tau}/T\in X_\tau\}$;

then we can write:

$$\{\underline{\tau}/T \in X_\tau\} \equiv \{\underline{\tau}_1 \in X_1,\ \underline{\tau}_2 \in X_2, \ldots,\ \underline{\tau}_s \in X_s\} \in \{<\underline{\tau}_1,\ \underline{\tau}_2, \ldots,\ \underline{\tau}_s>\} \equiv X_1 \times \ldots \times X_s,$$

where $X_1 \times \ldots \times X_s$ is the Cartesian set product. Thus any mapping $\{\underline{\tau}/T \in X_\tau\} \to d$ is equivalent to the set of simplest mappings from class K_I

$$\{<\underline{\tau}_1,\ \underline{\tau}_2, \ldots,\ \underline{\tau}_s> \to d \mid <\underline{\tau}_1,\ \underline{\tau}_2, \ldots,\ \underline{\tau}_s> \in X_1 \times \ldots \times X_s\}.$$

In the class K_{III} , any predicates can be defined on the set of test results. As distinct from the classes K_I и K_{II}, the class K_{III} models may be classified as nonprimitive models of knowledge because new functional parameters (e.g., coefficients in equations or inequalities) that do not belong to the test set $\{\tau/T\}$ may appear. Most pattern recognition models belong to the class K_{III} .

Any model k from the class K_I is representable as $k = k_a \cup k_p$ where

$k_a = \{\{\underline{\tau}/T\} \to d\}$;

$kp=\{\neg\{\tau/T\}_1 \& \ldots \& \neg\{\tau/T\}_m \to \neg d\} \cup (d_1 \vee \ldots \vee d_n,\ \neg d_1 \& \ldots \& \neg d_{n-1} \to d_n)$.

The component k_a may be called the active part of a model of knowledge while k_p plays a passive role because it is completely determined by the active part. As the mappings $\{\{\underline{\tau}/T\} \to d\}$, we will consider all irredundant mappings, i.e. mappings whose left parts are minimal combinations of test results which are sufficient to draw a conclusion from the available data (the example set R^+). For each conclusion $d_j \in D$ there exists a minimum set of irredundant mappings which in the aggregate cover all the examples from $R^+(d_j)$. Strictly speaking, for each conclusion there may exist more than one minimum set. The construction of minimum, irredundant models of knowledge in the class K_{II} is almost identical to the construction of the same models in the class K_I with one addition. Once all the simplest irredundant mappings have been set off, a convolution operation should be performed.

Notation τ/T implies that the results of the test τ can take the values of different domains T. Domains can represent a distinct level of generality. Consider the examples [3].

Assume that T1 – T4 are distinct domains for description of the human temperature:

T1 = [34, 42] degrees;

T2 = {[34, 35], (35, 36.5), [36.5, 36.8], [36.9, 37.4], [37.5, 40]};

T3 = [decreased temperature; normal temperature; elevated temperature; high temperature];

T4 = [normal temperature; abnormal temperature].

The above-mentioned groups of domains have the desired property that if the value of the test is given on one domain, values of the test may be determined on domains with a greater number by using the fixed (single) rules of recalculation.

In other words, by using the domains cited an improper order can be given by the criterion of generality (the relation of domination), namely:

T1 ≤ T2 ≤ T3 ≤ T4.

The rules by which the values from one domain are translated into another may be specified in different ways, for example, on the basis of fuzzy-set theory or using neural networks. By way of example, below are the simplest rules:

T2.{[34, 35], (35, 36.5)} → T3.{decreased temperature}; T2.{[36.5, 36.8]} → T3.{normal temperature};
T2.{[36.9, 37.4]} → T3.{elevated temperature}; T2.{[37.5, 40]} → T3.{high temperature};
T3.{normal temperature} → T4.{normal temperature};
T3.{decreased temperature; elevated temperature; high temperature} → T4.{abnormal temperature}.

If we replace the sign ‘≤’ with the implication sign ‘→’, then for the relation of domination we will obtain *an oriented graph of domains* with a single root node, which symbolizes the objective level (the minimum level of generality). The oriented graphs of all test domains are part of the nonprimitive ontology of the subject domain.

In the general case an oriented graph of domains can be determined for each test (factor). Any path on the graph implies a possibility of a unique recalculation of values from one domain to other one. One can set some graphs for any test.

In searching through all possible combinations of domains for diverse tests we derive a complete set of descriptions of reality with a variety of levels. We name such descriptions which cannot be generalized by one test without breaking the condition of separability as *critical* ones. Descriptions that can be generalized from at least one test without violating the separability condition will be termed *subcritical*. Descriptions that violate the separability condition will be termed *postcritical*. A set of optimal models of knowledge for all descriptions (subritical, critical and postcritical) forms a *complete model of multilevel description of reality*.

We name the model which allows of solving a target task for any presented situation of reality as *true* one. In so doing, a new situation of reality is generalized at most in the context of the complete model of multilevel description resulting in a simplifying of the solution.

Thus the principle of the method of limiting generalizations lies in the following:

1. The maximum branched graph of domains (or some graphs with different domination relation realization mechanisms) is built for each test involved in description of the task. Experts in the subject domain play a large role in the construction of graphs.

2. An optimal model of knowledge is built for each combination of domains defining the level of generality of description. A set of all optimal models of knowledge defines the complete model of a multilevel description of reality.

3. In searching the solution for a new situation a given situation is generalized at most to one of descriptions including a true model of knowledge (it is desirable to generalize to a critical description). The solution is situated at a new level of description. If the solution is not available, it is necessary to correct models of knowledge. It is important to keep in mind that the availability or the lack of the solution is dependent on a subjective estimate of truth of knowledge models (representativeness of samples at one or another level).

3 Conclusions

The application of this method opens up broad avenues for the development of intelligent information systems of various purposes, in particular, knowledge bases for hospital systems [2-10], telemedicine systems [10] and learning systems [3, 10]. Electronic patient or pupil/student records are used as a priori information.

Not only does the method allow a single agent (expert) to solve problems, but it also makes possible a variety of concilia of intelligent agents [8 - 10]. This possibility is provided by each agent having a multilevel reality description model (knowledge field). The multiplicity of reality description levels allows the agents to find compromise information interchange schemes, eventually providing the solution of complex problems.

References

[1] Prokopchuk Yu. A. Pattern and Wave Model of Intelligence // Artificial Intelligence, 2002.- № 3. - P.124-132. (In Russian)

[2] Prokopchuk Yu. A. Intellectual Medical Systems: Formal and Logic Level. - Dnepropetrovsk: Institute of Technical Mechanics, NASU & NSAU, 2007.-259 p. (In Russian)

[3] Alpatov A.P., Prokopchuk Yu. A., Yudenko O.V., Khoroshilov S.V. Information Technologies for Education and Public Health. - Dnepropetrovsk: Institute of Technical Mechanics, NASU & NSAU, 2008.-287 p. (In Russian)

[4] Alpatov A.P., Prokopchuk Yu. A., Kostra V. Hospital Information Systems. - Dnepropetrovsk: Ukrainian State University of Chemical Engineering, 2005.-257 p. (In Russian)

[5] Prokopchuk Yu, Kostra V. Representation of Algorithmic Knowledge in Medical Information Systems // 7th International Conference "Knowledge-Based Intelligent Information and Engineering Systems" (KES 2003) (Oxford, UK, September 3-5, 2003): Proceedings, Part II, Series: Lecture Notes in Artificial Intelligence, Vol. 2773. – Berlin: Springer-Verlag Berlin Heidelberg, 2003. - P.1069-1074.

[6] Prokopchuk Yu., Kostra V. Technology of construction of open local expert systems attached to his medical documents // IFMBE Proceedings, 2nd European Medical and Biological Engineering Conference (EMBEC'02) (Vienna, Austria, 2002). Vol. 1. - P.766-767.

[7] Prokopchuk Yu., Kostra V. Architecture of Hospital Information Systems // Proceedings of the 2001 International Conference on Mathematics and Engineering Techniques in Medicine and Biological Sciences (METMBS'2001) (Las Vegas, Nevada, USA, June 25-28, 2001) / Ed. Valafar. - Las Vegas: CSREA Press, 2001. - P. 197-200.

[8] Prokopchuk Yu. Doctors' concilia with the participation of intelligent systems // Bulletin of Kherson National Technical University, 2007. – No 4(27). – P. 198–202. (In Russian).

[9] Prokopchuk Yu. Consilium of Intellectual Systems // Artificial Intelligence, 2007. - №4.- C. 491 – 496. (In Russian).

[10] Prokopchuk Yu. Multipurpose Knowledge Bank in Clinical Medicine // Bulletin of Kherson National Technical University, 2008. – No 1(30). – P. 14–20. (In Russian).