ABOUT THE INTERVAL (SET) ANALYSIS OF THE PROCESSES

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1 Introduction

The values of all real processes are bounded. It does not correspond to the generally accepted probabilistic model of the non-determining processes when the normal distribution law of the probability is used for their values [1]. On the other hand when the process does not contain the constant component (it is centered) then from its boundary of the follows for the average arithmetical values on the finite intervals of the signal processing. Also the first difference of this process and its average arithmetical values on the finite intervals are bounded correspondingly. The essential operation during the primary processing is a smoothing for the process and its first difference by means of the glided windows with the chosen stated width because of the results of such smoothing are the bounded processes also [2].

The possibilities are considered for insertion of the interval characteristics of the unknown bounded signals [3-5], when the interval functions will use, in general, for estimation and prognostics of the values for the not fully predicted values of these signals.

For the non-determining processes for the conditions its indefinitely and unforeseeably it is expediently to take into account the parameters this boundary during the calculations of the statistical characteristics, the corresponding simulations and for obtaining the estimations of the corresponding information parameters for these processes [6, 7].

2 The interval characteristics of processes

To take into account the boundary of the values real processes for their modeling it is expediently to take into account such statistical characteristics where the parameters are figured of this boundary. In the classical models of processes the theory of probability these information is not taken in to account.

Let's consider the model of measurements or observations for such real process can be introduced as

$$y_n = x_n + f_n, \quad n \in (-\infty;\infty), \tag{1}$$

where y_n is the scalar digital sequence that is obtained as a result of measurements or observations, x_n is the unknown sequence of the true values for the measured or observed process, and f_n is the unknown bounded sequence that represents the measurement or observation errors. The processing procedure of the obtained values is based on this model for calculation of the estimations for the real process values. It depends on the received suggestions as to the measurent or observations noises essentially. In contrast to the probability approach let's consider the case when the interval characteristics of the bounded perturbations are used [3-5]. The assumption is simple and physically proved about the boundary of the errors (noises) and the speed of its modification [4]:

$$\left|f_n\right| \le \delta = const, \quad \left|\Delta f_n\right| \le \gamma = const\ \forall n, \quad \Delta f_n = f_{n+1} - f_n \,. \tag{2}$$

In other words the values of the digital sequence members f_n $(n \in (-\infty;\infty))$ are not predictable but they answer to the condition (2). At the same time it is supposed that f_n does not content the constant component (centered noise) that is only the noised component of the errors. At presence of the constant methodical error it is regarded to the unknown true values of the measurement process. Let's consider also the ability to smooth the noised component of the error and the velocity its modification by means of the glided windows with chosen and fixed width [2]. The simple glided window which is used for smoothing it is the rectangle window

$$N_0 = 2N + 1, \quad N = const \geq 0 \,. \tag{3}$$

Then for every value $N \geq 0$ of the smoothed sequence $\bar{f}_n$ it is determined by formulae

$$\bar{f}_n = N_0^{-1} \cdot \sum_{i=-N}^{i=N} f_{n+i} \quad \forall n, \quad N = 0,1,2,... \,. \tag{4}$$

In the other hand let's consider the smoothed sequence of the values for velocity of the error modification

$$\overline{\Delta f}_n = N_0^{-1} \cdot \sum_{i=-N}^{i=N} \Delta f_{n+i} \quad \forall n, \quad N = 0,1,2,... \,. \tag{5}$$

Obviously that according to (2)

$$\left|\overline{\Delta f}_n\right| \leq \gamma \ \forall n \,. \tag{6}$$

However, $\Delta f_{n-N} + \Delta f_{n-N+1} + \Delta f_{n-N+2} + ... + \Delta f_{n+N} =$

$$= f_{n-N+1} - f_{n-N} + f_{n-N+2} - f_{n-N+1} + f_{n-N+3} - f_{n-N+2} + ... + f_n - f_{n-1} +$$
$$+ f_{n+1} - f_n + f_{n+2} - f_{n+1} + ... + f_{n+N} - f_{n+N-1} + f_{n+N+1} - f_{n+N} = f_{n+N+1} - f_{n-N} \,.$$

In other words, instead of (5) it can be written

$$\overline{\Delta f}_n = N_0^{-1} \cdot \sum_{i=-N}^{i=N} (f_{n+N+1} - f_{n-N}) \quad \forall n, \quad N = 0,1,2,... \,. \tag{7}$$

Then according to (2), from (7) it is obtained the estimation

$$\left|\overline{\Delta f}_n\right| \leq 2\delta / N_0 \ \forall n \,. \tag{8}$$

Finally, it is obtained that inequality system is true

$$\Delta m_н(N) \leq \frac{1}{N_0} \sum_{i=-N}^{i=N} \Delta f_{n+i} = N_0^{-1} \cdot \sum_{i=-N}^{i=N} (f_{n+N+1} - f_{n-N}) \leq \Delta m_в(N) \ \forall n, N = 0,1,2,..., \tag{9}$$

where $\Delta m_н(N)$ and $\Delta m_в(N)$ are low and high boundaries of the interval function for estimation the average arithmetical value of the error modification velocity. In this case according to (6) and (8) the estimations are true for pointing boundaries

$$\begin{cases} \Delta m_н(N) \geq -\gamma, \quad \Delta m_в(N) \leq \gamma \quad \textit{for such } N \geq 0, \textit{that } \gamma \leq 2\delta/(2N+1), \\ \Delta m_н(N) \geq -2\delta/(2N+1), \quad \Delta m_в(N) \leq 2\delta/(2N+1) \quad \textit{for all other } N. \end{cases} \tag{10}$$

The inequality system, that is analogous to (9), is true and relatively to the values of the noise sequence. For this let's consider the notion of the integrating sequence $\tilde{f}_n$, that is the solution of the differential equation

$$\Delta \tilde{f}_n = f_n, \quad \tilde{f}_{n+1} = \tilde{f}_n + f_n \quad \forall n, \quad \tilde{f}_{n_0} = 0 \,, \tag{11}$$

where n_0 is some starting time moment from it the investigation is started for sequence f_n .

Let's ussume, that boundary is known

$$\left|\tilde{f}_n\right| \leq \sigma = const \quad \forall n \geq n_0 \,. \tag{12}$$

Then we obtained the inequality systems

$$m_н(N) \le \frac{1}{N_0} \sum_{i=-N}^{i=N} f_{n+i} = N_0^{-1} \cdot \sum_{i=-N}^{i=N} (\tilde{f}_{n+N+1} - \tilde{f}_{n-N}) \le m_в(N) \; \forall n \ge n_0, \; N = 0,1,2,..., \quad (13)$$

where $m_н(N)$ and $m_в(N)$ are $\Delta m_в(N)$ are low and high boundaries of the interval function for estimation the average arithmetical value of the error modification velocity correspondingly. For these boundaries the next estimations are true:

$$\begin{cases} m_н(N) \ge -\delta, \quad m_в(N) \le \delta \quad \text{for such } N \ge 0, \text{що } \delta \le 2\sigma/(2N+1), \\ m_н(N) \ge -2\sigma/(2N+1), \quad m_в(N) \le 2\sigma/(2N+1) \quad \text{for all other } N. \end{cases} \quad (14)$$

The estimations (14) and (10) can be used as boundaries of the interval function of the average arithmetical value for noise and the speed of its changing when the true boundaries in (13) and (9) are unknown. The boundaries can be estimated by the experimental way conducting the treatment of the sufficient long and representative realizations of the researched process. The notion of the representative realization is connected with the fact achieving of the values for this realization which are closed to the extreme values correspondingly to (2) and (12) even so many times.

Thus the interval characteristics (9) and (13) are obtained for the bounded centered process which are different essentially from the characteristic of the stochastic processes that have the type functions of the distribution for its values [1]. However the interval characteristics can be also entered for the bounded processes which generalize the known characteristics in the theory of the stochastic processes [3-5].

Let's assume that the restrictions (2) determine some set of the allowable values of the undefined sequence which let's name the set of the elementary events – F_O . Some choosing algorithm of these set elements forms the sequences of the numbers of some class. This algorithm is undefined completely. Then we can enter the interval characteristics of some class sequences according to [3]. The characteristics we name the chaotic because which are formed by means of such algorithm that is named chaotic also.

Let's enter the indicative function of the some $\xi \in [-\delta; \delta]$ and members of the bounded unknown sequences f_n

$$I(\xi, f_n) \equiv \begin{cases} 1 \quad at \quad f_n \le \xi, \\ 0 \quad at \quad f_n > \xi. \end{cases} \quad (15)$$

<u>The notion 1.</u> Let's of the restricted sequences $f_n, \quad n \in (-\infty; \infty)$ is formed by means of some chaotic choosing algorithm of the elements from some set F_O of the events accordingly to (2) and (12). For this sequence of the interval function with the lower and upper borders for distribution of the members is exist. These borders-functions are

$$1 \ge P_н(\xi, N) \ge 0, \quad 0 \le P_в(\xi, N) \le 1, \quad (16)$$

where $\xi \in [-\delta; \delta]$, and $N = 0,1,2,\ldots,$ are the sequent values that determines the width N_0 of the glide smoothing window according to (3). The inequality system is true for these functions and members of the chaotic sequence

$$P_н(\xi, N) \le \frac{1}{N_0} \sum_{i=-N}^{N} I(\xi, f_{n+i}) \le P_в(\xi, N) \; \forall \; n, \quad N = 0,1,2,\ldots. \quad (17)$$

By the theory of the probability process let's establish the indication when the chaotic sequence identify as stochastic [1].

<u>The notion 2.</u> If for all $\xi \in F_O$ the borders transitions are exist

$$\lim_{N \to \infty} P_н(\xi, N) = \lim_{N \to \infty} P_в(\xi, N) = P(\xi), \quad (18)$$

then such chaotic sequences we identify as stochastic and the having a single meaning distribution function of the chaotic sequences $P(\xi)$ let's identify as the distribution function of the stochastic sequences on the set of F_O .

As shown in [3-5], by integrating of the inequalities (17) with ξ on the interval $[-\delta; \delta]$ we obtain the inequalities of view (13), where

$$m_н(N) = \delta - \int_{-\delta}^{\delta} P_в(\xi, N) d\xi, \quad m_в(N) = \delta - \int_{-\delta}^{\delta} P_н(\xi, N) d\xi, \tag{19}$$

that is both types of the interval characteristics are interfaced. From (18) and (19) it follows that for the stochastic sequence is

$$\lim_{N\to\infty} m_н(N) = \lim_{N\to\infty} m_в(N) = m_0 , \tag{20}$$

where m_0 corresponds to the average of distribution in the theory of probability completely.

Thus the interval frequency function of the members of the undefined sequence can be established. By means of this function the full analog of the distribution density is obtained for the members of the stochastic sequence etc. [3-5].

Let the true values of the processes are depended on the some specified digital sequence of the inputs $u_n \ (n = \overline{1, M})$, in which connection this dependence is

$$x_n = \sum_{k=1}^{S} l_k \cdot \varphi_k(u_n, n), \quad n = 1,2,\ldots,M , \tag{21}$$

where $\varphi_k(\cdot)$ are the known functions of its arguments (so called "strong" functions), and $l_k \ (k = \overline{k = 1, S}, \quad S = const)$ are the unknown constant parameters. For convenience of the subsequent analysis the known functions are collected in the string-vector $\Phi_n = (\varphi_1(u_n, n); \varphi_2(u_n, n); \ldots; \varphi_S(u_n, n))$, and the unknown parameters are collected in the vector $L^T = (l_1; l_2; \ldots; l_S)$. To determine the estimations of the values of these parameters with the sufficient accuracy as a result of the processing M of the obtained values $y_n \ (n = \overline{1, M})$ the interval (set) analysis can be used [6, 7]. To obtain the authentic mathematical model of the concerned process in such a way the interval (set) analysis can be used also. It gives the possibility for its subsequent prediction. At the same time the next ratio is base for such processing

$$y_n = \sum_{k=1}^{S} l_k \cdot \varphi_k(u_n, n) + f_n, \quad n = 1,2,\ldots,M . \tag{22}$$

For using of the restrictions, which are connected, with velocity of the obstacle change the next ratios will be used

$$\Delta y_n = \sum_{k=1}^{S} l_k \cdot \Delta\varphi_k(u_n, n) + \Delta f_n, \quad n = 1,2,\ldots, M - 1 , \tag{23}$$

where $\Delta y_n = y_{n+1} - y_n$, $\Delta\varphi_k(u_n, n) = \varphi_k(u_{n+1}, n+1) - \varphi_k(u_n, n)$.

The presence of the noises does not give the possibility to obtain the accurate parameter values in the result of processing but it allows to obtaining their secure estimations in view of the set in the parameter space E^S as follows from boundaries (9) and (13). This parameter space is marked by inequalities which are obtained with the condition of the satisfaction of the ratios (9) and (13) for all n and at values of the parameters from these sets and for some values of f_n and Δf_n, that are satisfied to the these boundaries. If in the processing we obtain the empty set then it means that or the hypothesis about the connection of the true values to the "inputs" is not performed (through the uncorrected selection of view or quantities of "strong" functions), or the information about interval characteristics of the noises is inaccurate. This case we will not consider here.

Let's form the data processing procedure. For this we rewrite (22) in a view

$$y_n - \Phi_n \cdot L = f_n, \quad n = 1,2,\ldots, M . \tag{24}$$

Then we can write expression for the sum of any inequalities (22) successively that is divided to their quantity starting from some $(n - N)$ to $(n + N)$ equality.

$$\frac{1}{N_0}\sum_{i=-N}^{i=N}(y_{n+i}-\Phi_{n+i}\cdot L)=\frac{1}{N_0}\sum_{i=-N}^{i=N}f_{n+i}\ \forall\ N=0,1,2,...,(M-1)/2,\ \ n=N+1,N+2,...,M-N\ .$$

Hence, basing on (13), we obtain the system of the linier inequalities

$$\begin{gathered} m_н(N)\le\frac{1}{N}\sum_{i=-N}^{i=N}(y_{n+i}-\Phi_{n+i}\cdot L)\le m_в(N) \\ \forall\ N=0,1,2,...,(M-1)/2,\ \ n=N+1,N+2,...,M-N, \end{gathered} \tag{25}$$

that defines the set of the vector values L in the space of parameters E^S, which satisfy all inequalities (25) at the specified limits $m_н(N)$ and $m_в(N)$ of the interval function for estimation of the average arithmetical value of the noises. The system of the inequalities (25) can be rewritten in the compact view

$$\begin{gathered} \overline{y}(n,N)-m_в(N)\le\overline{\Phi}(n,N)\cdot L\le\overline{y}(n,N)-m_н(N) \\ \forall\ N=0,1,2,...,(M-1)/2,\ n=N+1,N+2,...,M-N, \end{gathered} \tag{26}$$

where the averaging-out values of the measured “inputs” and the selected “strong” functions.

The system of the linier inequalities can be write similarly to the system (25) based on (9) and (23)

$$\begin{gathered} \Delta m_н(N)\le\frac{1}{N}\sum_{i=-N}^{i=N}(\Delta y_{n+i}-\Delta\Phi_{n+i}\cdot L)\le\Delta m_в(N) \\ \forall\ N=0,1,2,...,(M-2)/2,\ \ n=N+1,N+2,...,M-N-1\ , \end{gathered} \tag{27}$$

It can be rewritten in the compact view

$$\begin{gathered} y_{n+N+1}-y_{n-N}-\Delta m_в(N)\le(\Phi_{n+N+1}-\Phi_{n-N})\cdot L\le y_{n+N+1}-y_{n-N}-\Delta m_н(N) \\ \forall\ N=0,1,2,...,(M-2)/2,\ n=N+1,N+2,...,M-N-1\ . \end{gathered} \tag{28}$$

Ex fact the set of the vector values L in the parameter space E^S, which contents the true values of the unknown parameters, is defined with the united system of the linier inequalities (26) and (28).

3 Conclusions

The method of the interval (set) analysis is used for obtaining the interval statistic characteristics of the undefined sequences (noises). The processing procedure of the measurement result with the bounded errors of the unknown process (estimating its parameters) is based on such characteristics.

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