

2.3 High-performance computing, including parallel and distributed computing

Perfect Vector Sequencing Theory and Its Applications for System Modeling

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Abstract. *This paper involves researches into systems engineering for improving the quality indices of technological, economical and others systems with respect to performance reliability and resolving ability, using novel design based on the Perfect Vector Sequencing theory. The ordered chain approach to the study of elements and events is known to be of widespread applicability for system modeling, when applied to the problem of finding the optimal arrangement of structural elements in a distributed information or technological system. We propose development of the scientific basis for technologically optimum distributed systems theory, namely the Perfect Vector Sequencing Theory, based on a new conceptual model of the multidimensional systems, and the generalization of the theory and combinatorial techniques to the improvement and structural optimization of larger class of technological problems.*

Keywords

System modeling, perfect vector sequencing theory, structural optimization, multidimensional system, technologically optimum distributed systems theory.

1 Introduction

Modern combinatorial design techniques and modeling are well using for finding optimal solution of wide classes of technological problems [1]. Especially we have many kinds of media whose architectures are described in terms of computer visualization [2]. However, the design based on the traditional combinatorial theory is not always applicable for multidimensional systems. In this connection a new approach to modeling the systems and processes is needed. In general case it was possible to take in consideration a conceptual model of the system as a sequence of numerical ordered -chain of sub-sequences to be of any length as well as number of terms in the sequence can be of any number too. Unfortunately, these numerical models are not very interest because its data redundancy as well as structural complexity. The problem, is known, to be of very important for configure multidimensional systems with fewer structural elements and bonds than at present, while maintaining or improving on resolving ability and the other operating characteristics of the system. Both advanced theory and regular method for finding optimal solution of the problem are needed.

2 Two-dimensional Ideal Ring Bundles

Let us regard the n -stage ring sequence $K_{2D} = \{(k_{11}, k_{12}), (k_{21}, k_{22}), \dots, (k_{i1}, k_{i2}), \dots, (k_{n1}, k_{n2})\}$, where we require all terms in each circular vector-sum to be consecutive 2-stage ($t=2$) sequences as elements of the sequence. A circular vector-sum is sum of consecutive terms in the ring sequence, which can have any of n terms as its starting point, and can be of any length (number of terms) from 1 to $n-1$. In addition, there is the sum of all n terms.

Hence the maximum number of distinct sums S_n of consecutive terms of the ring sequence is given by

$$S_n = n(n-1) + 1 \tag{1}$$

An n -stage ring sequence K_{2D} of the ordered 2-stage ring topology $\{(k_{11}, k_{12}), (k_{21}, k_{22}), \dots, (k_{i1}, k_{i2}), \dots, (k_{n1}, k_{n2})\}$ for which the set of the sums forms an ideal 2D grid (each 2D sum occurring exactly once) is called an “2D Ideal Ring Bundle” (2D IRB).

Here we consider an example of 2D IRB with four ($n=4$) two-dimensional vectors (2D terms) in the ring topology, where $k_1 = (1,1)$, $k_2 = (1,2)$, $k_3 = (1,4)$, $k_4 = (1,3)$, which graph is depicted below (Fig. 1).

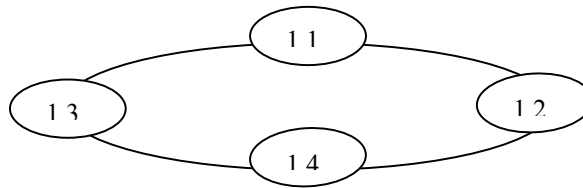


Fig. 1. Description of the 2D IRB with four ($n=4$) terms: (1,1), (1,2), (1,4), (1,3).

We can calculate easily all circular 2D sums, taking modulo $m_1 = 4$ for the first component of the 2D sum and modulo $m_2 = 5$ for the second its component:

$$\begin{aligned} (2,1) &\equiv (1,2)+(1,4) & (3,1) &\equiv (1,3)+(1,1)+(1,2) \\ (2,2) &\equiv (1,4)+(1,3) & (3,2) &\equiv (1,1)+(1,2)+(1,4) \\ (2,3) &\equiv (1,1)+(1,2) & (3,3) &\equiv (1,4)+(1,3)+(1,1) \\ (2,4) &\equiv (1,3)+(1,1) & (3,4) &\equiv (1,2)+(1,4)+(1,3) \end{aligned}$$

So long as the elements of the ring sequence themselves are also circular 2D sums, the circular 2D sums set will be as follows:

$$\begin{array}{cccc} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \end{array}$$

The result of the calculation forms the 3x4 grid, which exhausts the circular 2D sums and each of its meets exactly once. So, the ring sequence of the 2D terms $\{(1,1), (1,2), (1,4), (1,3)\}$ is two-dimensional Ideal Ring Bundle (2D IRB) with $n=4$ and $m_1=4, m_2=5$.

3 Multidimensional Ideal Ring Bundles

Multidimensional Ideal Ring Bundles (IRB)s of order n can be represented as n -stage ring-like sequence $K_{tD} = \{(k_{11}, k_{21}, \dots, k_{t1}), (k_{12}, k_{22}, \dots, k_{t2}), \dots, (k_{1n}, k_{2n}, \dots, k_{tn})\}$, which forms a set of circular t -dimensional sums on the sequence as $M_1 \times M_2 \times \dots \times M_t$ -matrix cycling exactly once. Fig. 2 shows a graphical interpretation of the Ideal Ring Bundle (t D IRB).

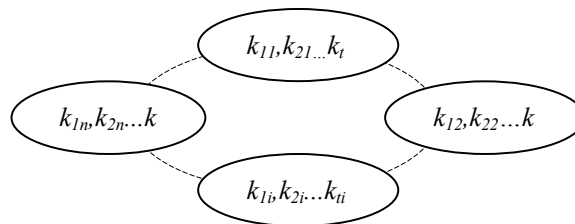


Fig. 2. Description of the t -dimensional IRB.

An n -stage ring topology sequence $\{s_1, s_2, \dots, s_i, \dots, s_n\}$ of terms, $s_i = \{k_{i1}, k_{i2}, \dots, k_{it}\}$, for which all circular t D sums of the sequence enumerate a set of $M_1 \times M_2 \times \dots \times M_t$ -matrix cells is called “ t -dimensional Ideal Ring Bundle” (t D IRB).

One of general formula for calculate of multidimensional Ideal Ring Bundles is given by equations:

$$n^2 - n + 1 = M_1 \times M_2 \times \dots \times M_t, \quad (2)$$

$$(M_1, M_2, \dots, M_t) = 1$$

As a result of research into the Perfect Vector Sequencing Theory it exists an infinite number of underlying combinatorial constructions.

4 Applications of the Perfect Vector Sequencing Theory

We can consider a t D IRBs as mathematical models of technologically optimum distributed systems, which make it possible to configure system with minimum numbers of elements and bonds while maintaining or improving on performance reliability, positioning precision and the other significant operating characteristics of the system.

4.1 Perfect Vector Data Coded System

Let us consider so-called "Monolithic Circular Binary Code" (MCBC). This code forms binary code combinations which all symbols "1" as well as symbols "0" are arranged together as being ring topology. We are interested in configure of vector data coded system, which all combinations of the MCBC exhaust a set of nodes of t -dimensional matrix described by equations (2). For example, the 2D MCBC formed on the two-dimensional (2D) Ideal Ring Bundle $\{(1,1), (1,2), (1,4), (1,3)\}$ is well illustrated by the next table, which follows from Fig. 1.

Tab.1. 2D MCBC coding system based on the Ideal Ring Bundle $\{(1,1), (1,2), (1,4), (1,3)\}$

(1,1): 1000	(2,1): 0110	(3,1): 1101
(1,2): 0100	(2,2): 0011	(3,2): 1110
(1,3): 0001	(2,3): 1100	(3,3): 1011
(1,4): 0010	(2,4): 1001	(3,4): 0111

Table 1 contains the set of binary code combinations for coding of all 2D vectors on the 4×3 grid from (1,1) (code combination 1000) to (3,4) (code combination 0111), where each of them has been coded in circular 2D MCBC. The main distinguish of the underlying code is that identical signals in each code combination to be consecutive elements of the combination. The remarkable property of the MCBC provides its some advantages over the rest codes. One of them is simplicity of error detecting and correcting as well as high-speed operation.

The maximum number of distinct sums S_n of consecutive terms of the ring sequence is given by (1). The perfect MCBC can be applied for compress a large array of image information, using innovative methodologies based on the remarkable properties and structural perfection of the multidimensional Ideal Ring Bundles.

4.2. Vector data logistic management

Next, we consider symbolic-form model of a vector data logistic management for a production flow process, designed on the 2D-IRB $\{(1,1), (1,2), (1,4), (1,3)\}$ which is shown in Figure3.

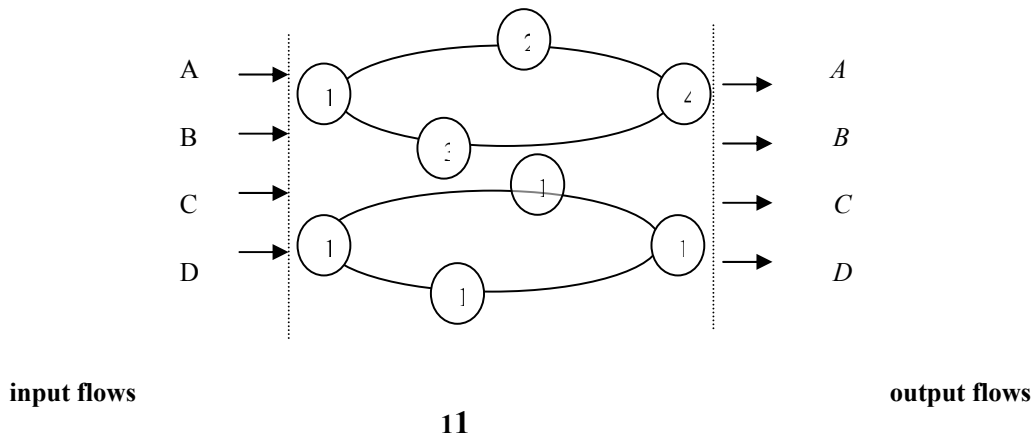


Fig. 3. A symbolic-form IRB model of vector data logistic management for a production flow process, designed on the 2D-IRB $(1,1), (1,2), (1,4), (1,3)$

Diagram of a circular manufacturing process considers four ($n=4$) incoming flows (A, B, C, D) and four output flows (A,B,C,D) to be arranged non-uniformly, so that cyclic relationship between points of its placement on the circle corresponds to the IRB symbolizes a concurrent enterprising program. The model provides any of cyclic operating or process as being 2D parameters, starting from programmed (one of four) place-time point, and finishing in one of the other its point. In general case both any of incoming flows (A,B,C,D) and any of engineering output flows (A,B,C,D) can be at any manner as well as at any time with the smallest possible number of the points.

Underlying model can be well useful for development of high-quality lean production based on total management [3]. These design technique make it possible to configure total quality management due to possibility of favourable distribution of in-line production with fewer total expenses for manufacturing process.

5 Conclusion

The Perfect Vector Sequencing Theory provides, essentially, a new conceptual model of technical systems, based on the idea of “perfect” combinatorial constructions, and the remarkable physical properties of space and time. The wonderful properties and structural perfection of multidimensional IRBs provide many opportunities to apply them for generalization of these models and methods for improvement and optimization of a larger class of technological systems, using methodologies based on combinatorial techniques. We better understand of the fundamental role of symmetry and non-symmetry in the behavior of natural and man-made objects as well as existing of perfection, harmony and beauty in the real world.

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